36. $\int_0^8 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^8 = \frac{2}{3}\left(9^{3/2} - 1^{3/2}\right)$

$= \frac{2}{3}(27-1) = \frac{52}{3}$

19. $\int_1^4 \sqrt{1+\frac{1}{4x}}\,dx$

$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$

$y = \int \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$= \frac{1}{2}\frac{x^{1/2}}{1/2} + c$

$= \sqrt{x} + c$

$1 = 1 + c \quad c = 0$

calc $\sqrt{\frac{1}{x}}\,x = \frac{\sqrt{x}}{\cancel{\sqrt{x}}\sqrt{x}}\frac{1}{}\cdot\cancel{x}$

Jan 22-8:57 AM

13. $x = \frac{y^3}{3} + \frac{1}{4y} \qquad y=1 \quad y=3$

$\frac{1}{4}y^{-1} \qquad -\frac{1}{4}y^{-2} = -\frac{1}{4y^2}$

$\frac{dx}{dy} = y^2 - \frac{1}{4}\frac{1}{y^2}$

$\int_1^3 \sqrt{1+\left(y^2 - \frac{1}{4y^2}\right)^2}\,dy$

$\int_1^3 \sqrt{1 + y^4 - 2\cdot\frac{1}{4} + \frac{1}{16y^4}}\,dy$

$\int_1^3 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}$

$\int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}$

$\int_1^3 y^2 + \frac{1}{4y^2}\,dy$

$= \frac{y^3}{3} + \frac{1}{4}(-y^{-1})\Big|_1^3$

$= \frac{y^3}{3} - \frac{1}{4y}\Big|_1^3$

$\left(\frac{3^3}{3} - \frac{1}{12}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)$

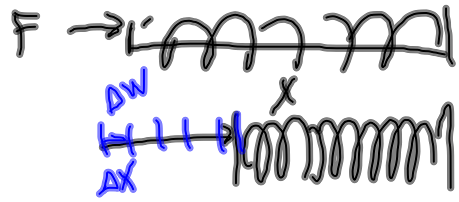
Jan 22-9:34 AM

7.5 work

if force is constant, $W = F \cdot d$ (force, distance)

if force varies over distance $F(x)$
(force is a function of distance)

then $\boxed{W = \int_a^b F(x)\,dx}$

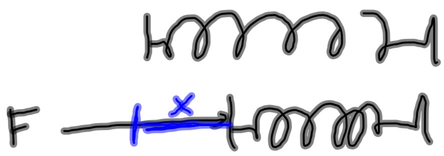


$\Delta W = F_i \,\Delta X$

$W = \lim\limits_{\Delta X \to 0} \sum F_i \,\Delta X$

Jan 22-9:42 AM

spring rest equilibrium length



$F = kx$

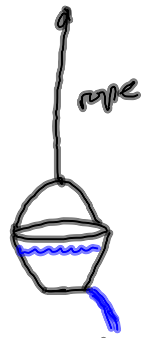
Hooke's law

apply $F = 50$ N to compress
a spring thru a distance of .2 m
How much work is done?

1. Find k $50 = k(.2)$ $k = \frac{50}{.2} = 250$
2. Find W $\int_0^{.2} 250x\,dx = 5$ Nm $= 5$ j

Jan 22-9:51 AM

Ex 2



bucket $22N \cdot 20\,m = 440\,j$

water (linear) $x=0$ $F=70$

x = distance bucket has been lifted

$x=20$ $F=0$

rope $20\,m\ (.4\ \frac{N}{m})$

$x=0$ $F=8\ N$

$x=20$ $F=0$

slope $= \frac{0-8}{20-0} = -\frac{8}{20} = -\frac{2}{5}$

$$\int_0^{20} -\frac{2}{5}(x-0)+8\,dx = 80\,j$$

$m = \frac{0-70}{20-0} = -\frac{7}{2}$

$F = -\frac{7}{2}(x-0)+70$

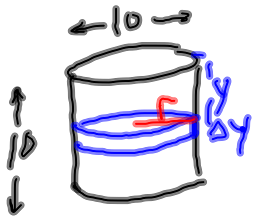
pt. slope $y = m(x-x_1)+y_1$

$F = -\frac{7}{2}x+70$

$$\int_0^{20} -\frac{7}{2}x+70\,dx = 700\,j$$

Jan 22-9:56 AM

pumping water out of a tank



ΔW = work for 1 slice

$= y\,\Delta F$

$$\int_0^{10} 62.4\pi 5^2\, y\, dy$$

245,044 ft-lbs

ΔF = weight of slice

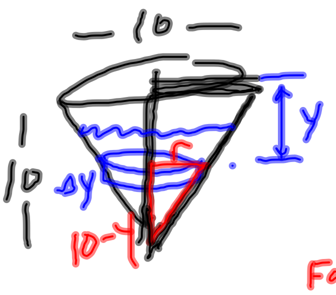
density of water $= 62.4\ \frac{lb}{ft^3}$

$\Delta F = \Delta V \cdot 62.4$

$r=5$ $\quad = \pi r^2 \Delta V (62.4)$

$\Delta F = 62.4\pi 5^2 \Delta y$

Jan 22-10:10 AM



$$\int_0^{10} 62.4\,\pi\left[\tfrac{1}{2}(10-y)\right]^2 \cdot y\,dy$$

Force = weight , distance

$$\Delta W = \underbrace{62.4\ \pi r^2\,\Delta y}_{\text{density}\,\cdot\,\text{volume}} \cdot y$$

r varies, need r as a function of y

similar Δ

$$\frac{5}{10} = \frac{r}{10-y} \qquad r = \frac{1}{2}(10-y)$$

EX 3 filled to within 2 ft of top with olive oil

$$\int_{-2}^{10} 57\,\pi\left(\tfrac{1}{2}(10-y)\right)^2 y\,dy$$

Jan 22-10:20 AM